Numerical algorithm for solving the Craig's model

Craig's model states that population size n can be found from mark-recapture data as

$$\ln(n) - \ln(n - r) = \frac{s}{n}$$

where r is number of captured individuals and s is number of captures. This problem is equivalent to finding a root of function

$$y(n) = \ln(n) - \ln(n - r) - \frac{s}{n}. \tag{1}$$

Some characteristics of the function $y(n)$ can be found by performing a simple mathematical analysis:

- $y(n)$ is defined for $n > r$,
- $\lim_{n\to r+} y(n) = +\infty$,
- $y(n)$ has its minimum at $n = \frac{sr}{s-r}$,
- $y(n)$ decreases for $r < n < \frac{sr}{s-r}$,
- $y(n)$ increases for $n > \frac{sr}{s-r}$,
- $\lim_{n\to+\infty} y(n) = 0$.

Above written characteristics imply that function $y(n)$ has only one root and that this is located somewhere in interval $r < n < \frac{sr}{s-r}$. Function $y(n)$ is plotted in figure below for $r = 15$ and $s = 50$.

Since there is only one root of $y(n)$, bisection method is suitable for finding the root. Let us take two points, n_L and n_R, which satisfy relations $y(n_L) > 0$ and $y(n_R) < 0$, respectively. Obviously, the root of $y(n)$ lies between points n_R and n_L. These points may be chosen for example as $n_L = r + 10^{-9}$ and $n_r = \frac{sr}{s-r}$. Bisection method for root finding is an itterative one, single itteration consists of following steps:

1. Define point n_C lying in the center of interval $(n_L; n_R)$, i.e., $n_C = \frac{n_L+n_R}{2}$.
2. Compute function value $y(n_C)$.
3. Depending on the value of $y(n_C)$, redefine points n_L or n_R according to following conditions:

- If $y(n_C) > 0$, root must lie to the right from n_C: set $n_L = n_C$, keep n_R.
- If $y(n_C) < 0$, root must lie to the left from n_C: set $n_R = n_C$, keep n_L.

A numerical algorithm for root finding is based on repeating steps 1-3 until the size of interval $n_R - n_L < \epsilon$, where ϵ is some preset limit of accuracy. When the itterative procedure terminates, root estimate $n_{root} = \frac{n_L+n_R}{2}$ differs from actual root by less than $\frac{\epsilon}{2}$. Value of ϵ is set to 0.01.

Standard deviation of the result n_{root} is given as (see relation (9) in Craig's paper)

$$\sigma = \sqrt{\frac{1}{n_{root}(\exp(\frac{s}{n_{root}}) - 1 - \frac{s}{n_{root}})}}.$$

Ref.: C. C. Craig, *On the utilization of marked specimens in estimating populations of flying insects*, Biometrika **40**(1953), 170-176.

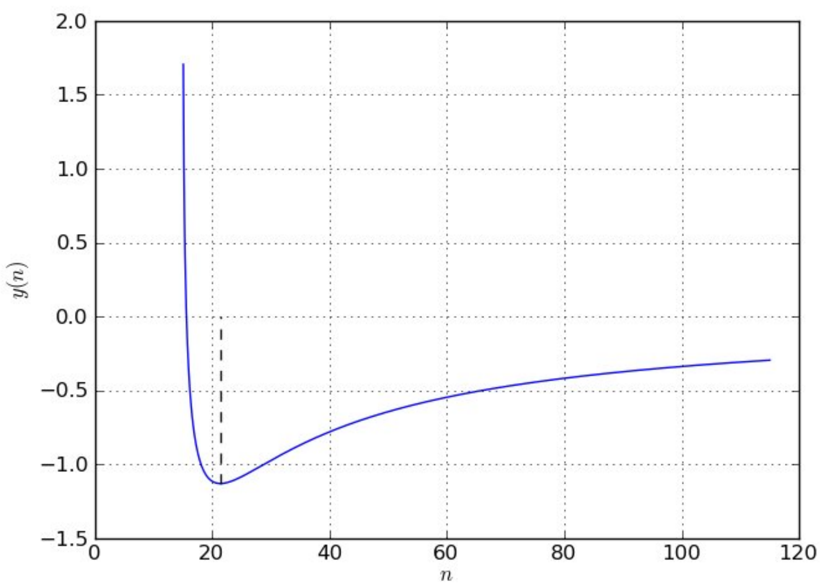

Figure 1: Function $y(n)$ (relation (1)) for $r = 15$, $s = 50$. Minimal value is located at $n = \frac{sr}{s-r} \approx 21.4$.